A comparative study of higher order Bragg waveguide gratings using coupled–mode theory and mode expansion modeling

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I. Introduction

LAMELLAR diffraction gratings integrated into slab waveguides (so–called Bragg waveguide gratings) found widespread applications in distributed–feedback (DFB) and distributed Bragg–reflector (DBR) lasers, either as edge or as surface emitters. The propagation of electromagnetic waves in these structures is essentially of two–dimensional nature. Furthermore, the finite length of the gratings disturbs the one–dimensional periodicity. During the last years, higher order Bragg waveguide gratings have attracted again increasing interest because of the demand for high–power DFB and DBR lasers emitting at wavelengths below 1 μm. Due to their larger periods, they can be easier fabricated.

For the numerical simulation of these structures, a large variety of different models exists. Many models are based on the coupled–mode theory (CMT), which was firstly applied to DFB lasers in [1] and which was later improved in [2]. In the CMT the mode field is expressed as an infinite summation of partial waves dictated by the Bloch–Floquet theorem. The result are coupled–mode equations for the amplitudes of the oppositely propagating waves directly interacting with the Bragg grating and an infinite set of equations for partial waves describing the radiation fields. As outlined in [2], these radiation fields lead to a modification of the coefficients entering the coupled–wave equations of Ref. [1].

Other models do not rely on the Bloch–Floquet theorem, but are based on a direct solution of Maxwell's equations. Some of these models based on the method of lines (MoL), bi-directional mode expansion or finite–difference time–domain (FDTD) methods were compared in [3] by calculating the modal reflectivity and transmittance of deeply etched short Bragg waveguide gratings. A good mutual agreement was found.

Despite this success, a comparison of these models with those based on the CMT is still missing. In this paper, the modal reflectivity and transmittivity of Bragg waveguide gratings are calculated using models based on the CMT [4] and the bi–directional mode expansion model CAMFR [5]. Thereby, an approximate solution for the partial waves firstly presented in [6] exploiting the free–space Green's function is corrected and generalized.

II. Benchmark example

The Bragg waveguide grating under study schematically drawn in Fig. 1 consists of a periodic stack of two slab waveguides, named slab 1 and slab 2. The lengths a and b of slab 1 and 2, respectively, are calculated from the duty cylce $D = a/\Lambda$ and the total length of the period of the grating $\Lambda = a + b$.

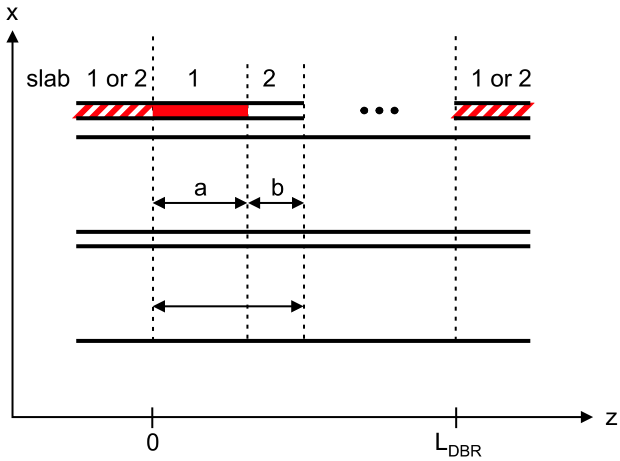

Fig. 1. Benchmark example

The DBR stack can be surrounded either by slab 1 and or by slab 2. The total length of the grating is about $L_{\mathrm{DBR}} \approx 200$ μm and the number of periods is given by the integer quantum of L_{DBR}/Λ. For a first order grating ($N = 1$), the number of periods is 1315.

The aim of the modeling task is the computation of the reflectivity R at $z = 0$ of the fundamental guided TE mode and the corresponding value for the loss $L = 1 - R - T$ with T being the transmittivity at $z = L_{\mathrm{DBR}}$ versus the duty cycle D for Bragg orders $N = 1 \cdots 3$.

III. Methods

The coupled–mode equations for the amplitudes of the oppositely going waves can be written as

$$\pm\frac{\partial\psi^{\pm}}{\partial z} = -i\Delta\beta\psi^{\pm} - i\kappa\psi^{\mp} \qquad (1)$$

for a symmetric grating. The complex valued total coupling coefficient κ is composed of terms due to the direct and indirect interactions of the two oppositely going waves with the grating [2], [4]. The indirect interaction gives also rise to a modification of both the real and imaginary parts of the relative propagation coefficient $\Delta\beta$ via the self–radiation coupling coefficient κ_s^r,

$$\Delta\beta = \frac{2\pi n_{\rm eff}}{\lambda} + i\frac{g-\alpha}{2} - \frac{N\pi}{\Lambda} + i\kappa_s^r \quad (2)$$

where $n_{\rm eff}$ is the effective index of the reference waveguide which is obtained by averaging the dielectric function $\varepsilon(x,z) \equiv n^2(x,z)$ along z. The modal gain–loss function $g-\alpha$ of the reference waveguide is set to zero for the benchmark example.

The inhomogeneous differential equations for the partial waves ares solved by means of the Green's function method using the one–dimensional Green's function of either a multilayer waveguide [4] and or a homogeneous unbounded medium. For the latter case explicit expressions for the coupling coefficients and radiation losses for arbitrary Bragg orders will be given.

In bi–directional mode expansion methods such as used in the tool CAMFR [7] the field in each slab is expanded onto eigenmodes in that slab. At the interfaces between the slabs, the transitions conditions give rise to reflection and transmission matrices. The propagation from one interface of the slab to the other interface results in diagonal propagation matrices.

For the determination of R and T of the entire structure, different algorithms can be used. The T–matrix algorithm (also known as transfer matrix method) is the mathematically simplest algorithm. But it is numerically unstable due to the combination of falling and growing exponentials. The derivation of the S–matrix algorithm which relates the incoming fields and the outgoing fields by means of scattering matrices is mathematically more involved. However, this algorithm can be implemented unconditionally stable. Similar holds for the R–matrix algorithm, but its implementation requires special treatment.

In CAMFR, the S–matrix algorithm is used for the calculation of R and T. The structure is terminated by an electric wall on top and bottom. In order to reduce the parasitic reflections caused by these boundary conditions, perfectly matched layers (PMLs) are placed in front of the walls implemented using complex coordinate stretching. For the simulation of the benchmark example, 100 TE eigenmodes were used and the thicknesses of the PMLs have been chosen to 2 μm with an imaginary part equals -0.5 μm.

IV. Results

Fig. 2 depicts examplary results for the maximum reflection coefficient and the corresponding values for the radiation loss coefficient versus duty cycle of a third order grating ($N = 3$). The dependence of the maximum reflectivity versus duty cycle, e.g., the position and values of maxima and minima and the increase is well reproduced by all models. There is also a surprisingly good correspondence in the radiation losses between CAMFR and CMT as well as between the two CMT models. The position of the maxima and minima of the loss is again well reproduced by all models. It should be noted, that the loss computed with CAMFR was corrected by the insertion loss. Details will be given at the conference. The small deviations between CAMFR and the CMT models are possibly caused partially by residual reflections, partially by the approximations inherent to the CMT models.

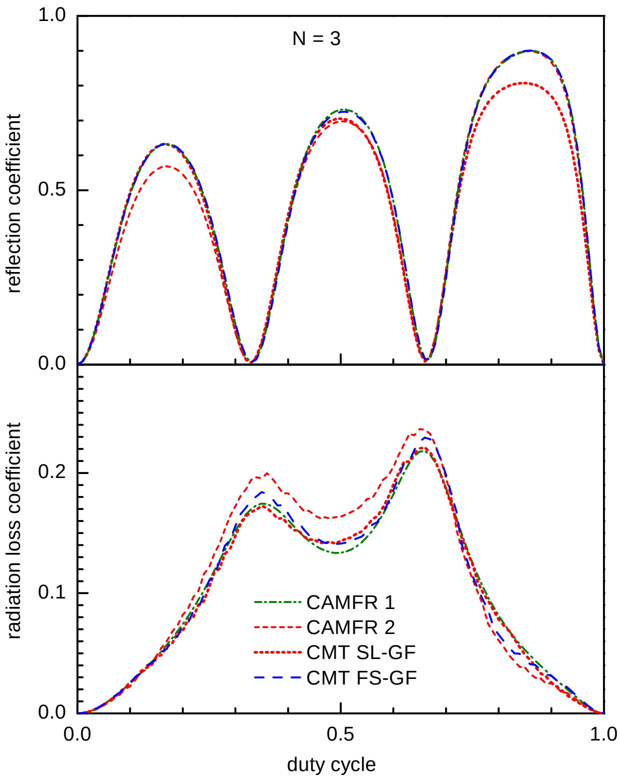

Fig. 2. Maximum modal reflectivity (top) and corresponding modal radiation loss (bottom) versus duty cycle for third order grating. Short dashed (CAMFR 1): Mode expansion method with slab 1 enclosing the Bragg waveguide grating. Dotted (CAMFR 2): Mode expansion method with slab 2 enclosing the Bragg waveguide grating. Dash–dotted (CMT SL–GF): Coupled–mode theory using slab Green's function. Long dashed (CMT FS–GF): Coupled–mode theory using free–space Green's function.

Acknowledgement

The authors would like to thank A. M. Shams-Zadeh-Amiri for participating in the modelling task and P. Bienstman for providing the CAMFR tool.

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