A New Multiport Measurement-Method Using a Two-Port Network Analyzer

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***Abstract*— A new method is presented how to characterize multiport devices using a two-port vector network-analyzer (VNA). Up to now, at least one of the port terminations had to be fully known to measure the S-parameters of the device. Our new measurement method overcomes this restriction. All of the device parameters and all of the port terminations are calculated from the device measurements.**

***Index Terms*— Multiport circuits, Millimeter wave measurements, Measurement errors, Scattering parameters measurement.**

I. Introduction

Although three and four port vector networks analyzers (VNAs) are commercially available [1], [2] and calibration routines for Multiport VNAs are well known [6], the two-port VNA is most common. Therefore, one often encounters the problem to characterize a multiport using a two-port VNA. Generelly speaking, to measure an N-port device with a two-port VNA, $N(N-1)/2$ two-port measurements have to be performed. If the remaining ports of the device are terminated by perfect loads, the S-parameters of the device can be deduced (with some redundancy) directly from the measurements.

However, perfect loads cannot be realized, especially for on-wafer measurements. Therefore, the measurement data has to be corrected for the errors introduced by the imperfect terminations. Several algorithms solving this task have been presented during the last years [3], [4], [5].

All of these methods are based on the idea to normalize the measured S-parameters to a port impedance equal to the termination connected to the particular port, when it is not connected to the VNA. To carry out this normalization, the terminating impedances have to be known a priori and, thus, have to be measured additionally.

For a three-port device, for example, three two-port measurements of the device and three one-port measurements of the port terminations have to be conducted. This results in a total of 12 unknowns (9 S-parameters and 3 terminations) requiring a measurement of 15 knowns.

In [5] it is shown that it is sufficient to know only one of the port terminations. The others can be calculated based on the measurement redundancy. Thus, the 12 unknowns are determined using 13 knowns.

But, in case of on-wafer measurements even the determination of only one of the loads is rather difficult. This is because the two probes cannot be connected directly and the necessary thru line between them shifts the reference plane. Leaving the unused ports unconnected during the measurements, it is more than ever impossible to characterize this imperfect open-stub correctly.

In this paper, we describe a method to determine one of the port terminations by an additional one-port measurement of the device, with all the remaining ports being terminated by their particular load impedances. This impedance can be an open as well so that it is even possible to determine the S-parameters of a multiport device, when leaving all unused ports unconnected.

The paper is organized as follows: Section II presents the derivation of the algorithm. Section III shows some results of on-wafer three-port measurements.

II. The Algorithm

For describing our method we use a three-port device measurement as an example. The unknown 3×3 matrix of the device is denoted as S. Three two-port measurements are performed, represented by the matrices S_a, S_b, and S_c, respectively. The three port terminations are denoted by their port number: r_1, r_2 and r_3. In order to simplify description, we make use of the cofactors of the 3×3 matrix S of the device:

$$C_{ij} = (-1)^{i+j}\,|S_{ij}|, \tag{1}$$

with $|S_{ij}|$ being the determinant of the 2×2 matrix obtained by deleting row i and column j in S.

For the first measurement, denoted by index a, port 3 is terminated by reflection coefficient r_3 and port 1 of the VNA is connected to port 1 of the device. The measured two-port S-parameters read:

$$S_a = \frac{1}{1-r_3S_{33}}\begin{bmatrix} S_{11}-r_3C_{22} & S_{12}+r_3C_{21} \\ S_{21}+r_3C_{12} & S_{22}-r_3C_{11}\end{bmatrix} \tag{2}$$

Similar expressions are obtained for the second and third measurements:

$$S_b = \frac{1}{1-r_2S_{22}}\begin{bmatrix} S_{11}-r_2C_{33} & S_{13}+r_2C_{31} \\ S_{31}+r_2C_{13} & S_{33}-r_2C_{11}\end{bmatrix} \tag{3}$$

$$S_c = \frac{1}{1-r_1S_{11}}\begin{bmatrix} S_{22}-r_1C_{33} & S_{23}+r_1C_{32} \\ S_{32}+r_1C_{23} & S_{33}-r_1C_{22}\end{bmatrix} \tag{4}$$

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Measurement b is performed by terminating port 2 with r_2 (port 1 of the VNA is connected with port 1 of the device again) and for measurement c port 1 is terminated with r_1 (with port 1 of the VNA connected to port 2 of the device).

For error correction, the measured S-parameters S_i, $i = a, b, c$, are now transformed into Gamma-R parameters as shown in [4]. This transformation is in principle the same as the transformation from [3], but it holds also for ideal shorts and opens as terminating impedances:

$$R_i = \left(\Gamma_i^* + S_i\right)\left(1 - \Gamma_i S_i\right)^{-1}, \tag{5}$$

where 1 represents the matrix of unity and Γ_i denotes a diagonal matrix of the particular terminations. For the order of measurements and the notation chosen here (which, however, is arbitrary) the Γ_i read:

$$\Gamma_a = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix},\ \Gamma_b = \begin{bmatrix} r_1 & 0 \\ 0 & r_3 \end{bmatrix},\ \Gamma_c = \begin{bmatrix} r_2 & 0 \\ 0 & r_3 \end{bmatrix} \tag{6}$$

Now, the three-port Gamma-R parameters R can be composed by combining the appropriate elements of R_i:

$$R = \begin{bmatrix} R_{a11} & R_{a12} & R_{b12} \\ R_{a21} & R_{c11} & R_{c12} \\ R_{b21} & R_{c21} & R_{b22} \end{bmatrix} \tag{7}$$

The reflection coefficients of R, i.e., the elements in the main diagonal, are overdetermined. E.g., instead of R_{c11} also R_{a22} could be used. This redundancy can be cast into three equations:

$$R_{11} \quad : \quad R_{a11} = R_{b11} \tag{8}$$

$$R_{22} \quad : \quad R_{c11} = R_{a22} \tag{9}$$

$$R_{33} \quad : \quad R_{b22} = R_{c22} \tag{10}$$

Unfortunately, these three equations cannot be solved for the three unknown terminations, because they are not linearly independent, but it is possible to write two of them as a function of the remaining one:

$$r_1 = \frac{r_2(|S_c| - S_{c11}S_{b22}) + S_{b22} - S_{c22}}{|S_b| + r_2(|S_c|S_{b11} - |S_b|S_{c11}) - S_{b11}S_{c22}} \tag{11}$$

$$r_3 = \frac{r_2(|S_a| - S_{b11}S_{a22}) + S_{b11} - S_{a11}}{|S_b| + r_2(|S_a|S_{b22} - |S_b|S_{a22}) - S_{a11}S_{b22}} \tag{12}$$

Now, with this solution, we are as far as in [5]. One of the three terminations (here r_2) must be known and then all of the unknowns can be calculated.

The problem is how to determine the remaining unknown r_2 without measuring it. Therefore, we measure the three-port device again, but at this time we connect two ports with their corresponding terminations. With port 2 terminated by r_2 and port 3 by r_3, the reflection coefficient S_{d11} reads:

$$S_{d11} = \frac{S_{11} - r_2 C_{3,3} - r_3 C_{2,2} + r_2 r_3 |S|}{1 - r_2 S_{22} - r_3 S_{33} + r_2 r_3 C_{1,1}} \tag{13}$$

Again, the cofactor and determinant formulation $C_{i,i}$ and $|S|$, respectively, can be used to simplify the expression. Normalizing S_{d11} to Gamma-R parameters, one obtains R_{d11} and, therefore, another equation for R_{11}:

$$R_{11} \quad : \quad R_{a11} = R_{d11} \tag{14}$$

This can be solved for r_2 and results in:

$$r_2 = \frac{S_{a11} - S_{d11}}{|S_a| - S_{d11}S_{a22}} \tag{15}$$

With (15), r_2 can be calculated from device measurements. Then, (11) and (12) can be used to calculate the two remaining unknown terminations r_1 and r_3, respectively.

Now all of the three terminations are known, and the three-port Gamma-R parameters from (7) can be transformed back into S-Parameters as shown in [4]:

$$S = \left(1 + R\Gamma\right)^{-1}\left(R - \Gamma^*\right), \tag{16}$$

where Γ represents the port impedances of the three-port:

$$\Gamma = \begin{bmatrix} r_1 & 0 & 0 \\ 0 & r_2 & 0 \\ 0 & 0 & r_3 \end{bmatrix} \tag{17}$$

III. EXAMPLE & VERIFICATION

For verification, we performed an on-wafer measurement of an MMIC Wilkinson power divider on GaAs substrate. The divider is designed for a frequency of 23.55 GHz in a coplanar environment. The circuit schematics is shown in Fig. 1. Port 2 denotes the input port of the divider, port 1 and 3 are the symmetric output ports.

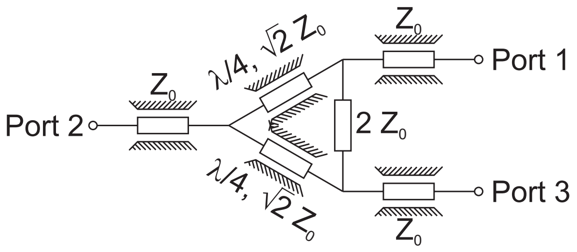

Fig. 1. Circuit schematics of the measured Wilkinson power divider.

Two different measurement cycles were performed. The first one, denoted with index I in the following, is used to demonstrate the presented algorithm. The particular unused port of the divider was left open. Three two-port measurement and the additional one-port measurement as described before with two open ports were performed. Then, the S-parameters of the device are calculated as shown in Section II. Because no termination for any of the port is needed, only two probe tips are used during this measurement.

To validate the results, a second measurement cycle (index II) was performed, where a 50 Ω load, realized by

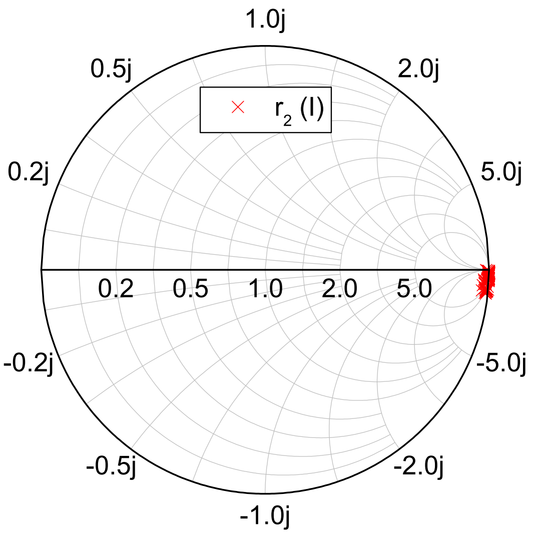

Fig. 2. Termination r_2 as extracted from measurement cycle I.

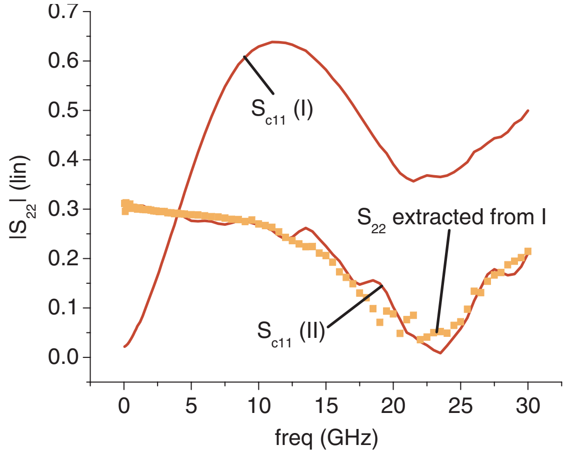

Fig. 4. Reflection coefficient $|S_{22}|$ of the power divider against frequency; lines denote the third measurements (index c) from cycle I and II, symbols denote the S_{22} value extracted from the measurements of cycle I by means of the new algorithm.

a 20 dB attenuator, was used to terminate the respective remaining third port. For this purpose, a third probe tip had to be installed at the probe station. Three two-port measurements were performed. They are used as a reference in the following.

In Fig. 2, r_2 as extracted from measurement cycle I is plotted. It can be seen that it represents an open as expected, with a small phase deviation because of the line extension of the coplanar open stub. The extractions for r_1 and r_3 look quite similar, so for the sake of clarity, they are not inserted in Fig. 2.

The output reflection coefficient (S_{11}, with the port notation from Fig. 1) is shown in Fig. 3. To point out the effect of the algorithm, the magnitude of the measured S_{b11} from the two measurement cycles are plotted, together with the extracted S_{11}.

For the measurement cycle II, S_{a11} looks quite similar to S_{b11} from Fig. 3, what indicates the quality of the termination used. Therefore, it is not included in Fig. 3. In comparison, S_{b11} from measurement cycle I is completely different from that of cycle II. This is because of the influence of the open connected at port 2 of the divider as calculated in eqn. (3). Looking at the extracted S_{11} reveals capability of the algorithm. Good agreement to S_{b11} of cycle II is observed, it even looks more smoothly.

Similar results are obtained for the input reflection coefficient S_{22} and the output port transmission S_{13} of

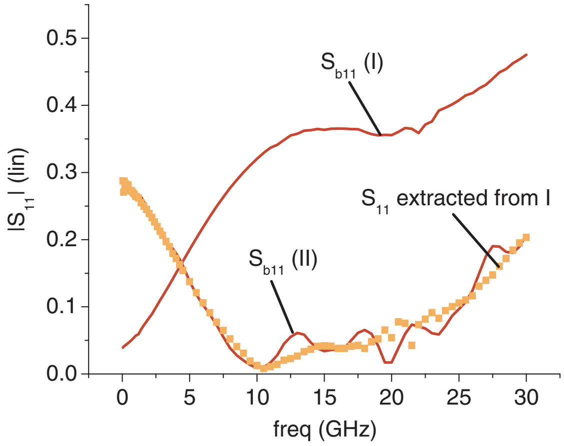

Fig. 3. Reflection coefficient $|S_{11}|$ of the power divider against frequency; lines denote the second measurements (index b) from cycle I and II, symbols denote the S_{11} value extracted from the measurements of cycle I using the new algorithm.

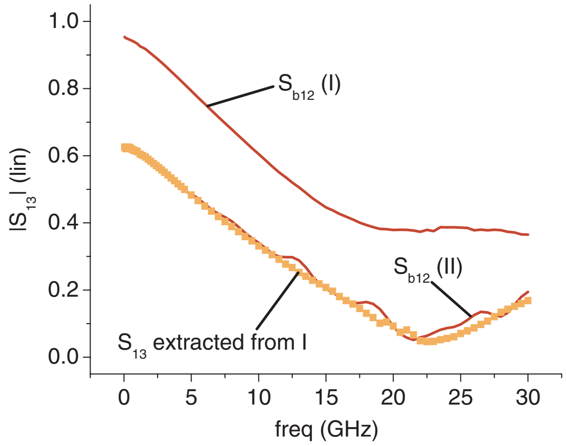

Fig. 5. Transmission coefficient $|S_{13}|$ of the power divider against frequency; lines denote the second measurements (index b) from cycle I and II, symbols denote the S_{13} extracted from the measurements of cycle I using the new algorithm.

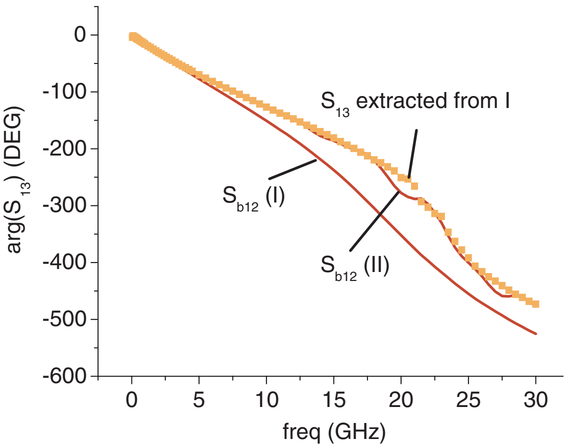

Fig. 6. Phase of S_{13} of the power divider against frequency; lines denote the second measurements (index b) from cycle I and II, symbols denote the S_{13} extracted from the measurements of cycle I using the new algorithm.

the divider, as shown in Figs. 4 and 5 and for the phase of S_{13} in Fig. 6, respectively. Good agreement between extractions and measurements of cycle II was found, although the measured curves from cycle I look completely different to the ones from cycle II.

Summarizing, it can be stated that the algorithm corrects the measurements error well, even for highly reflective terminations as the opens used in cycle I.

IV. CONCLUSIONS

A new algorithm for the measurement of multiports using a two-port VNA is presented. It uses an additional reflection coefficient measurement of the device with all of the remaining ports being terminated by their respective impedances. With this additional measurement all S-parameters of the device and all terminations can be extracted. This can be used either to simplify the measurement procedure or to increase accuracy.

The new method can be employed to simplify measurements when using a simple two-port VNA and letting the unused ports of the device unconnected. Then (more or less) stable opens are present at the device ports. They can be extracted as was shown in the example in Section III.

On the other hand, the new method can be applied in order to improve accuracy. Then a measurement setup is to be used, where all of the respective ports are connected to a specific termination (e.g. by means of a relays-switched two to three-port extension). Given this arrangement, the port terminations can be extracted more precisely, because they are measured when connected to the device.

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