Optimizing MMIC Reflection-Type Oscillators

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Abstract— **In this paper, optimization of the loaded quality factor Q_L for reflection-type HBT oscillators is investigated. Main result is an optimum relation between the S parameter phases at the three transistor ports. To support this finding, a 35.5 GHz MMIC VCO phase-noise better then -85 dBc/Hz at 100 kHz offset frequency is demonstrated.**

Keywords—Q **factor, Microwave oscillators, MMIC oscillators, Voltage controlled oscillators, Heterojunction bipolar transistors**

I. Introduction

Low phase noise oscillators are key components in each microwave system. One of the circuit concepts most commonly applied is that of the reflection oscillator. For this type, two ports of the active device are terminated by impedances in such a way that a negative resistance appears at the remaining port. There, a third impedance is connected to ground to adjust the oscillation frequency. In the following, these three impedances are referred to as external impedances. To ensure oscillation start-up, the product of the reflection coefficients from the active and the passive part of the circuit must be larger than unity with zero phase.

In the case of monolithic integrated circuits (MMICs), all three external impedances are comparable in magnitude, phase, and quality factor Q. Moreover, the inner elements of the transistor influence the oscillation frequency, and, obviously, they also contribute to the loaded quality factor Q_L of the oscillator. Therefore, it is not easy to identify the resonator of the circuit unambiguously and thus to determine Q_L. This complicates design for low phase noise, which, according to Leeson's well-known paper[1], should maximize Q_L of the oscillator.

Only few contributions on these MMIC-specific issues are available in the literature. Recently, Nallatamby et al.[2] published a first approach to generalize Leeson's treatment to the typical MMIC situation. The purpose of this paper is to demonstrate, how the optimum Q_L for the reflection-type HBT oscillator can be achieved. First, we establish a closed-form solution to calculate the loop gain and the loaded Q for this oscillator type. Afterwards, this approach is used to analyze the oscillator's properties as a function of the three external impedances.

The method is derived for a two finger 3x30μm^2 InGaP/GaAs HBT. The results are used to design and realize a 35.5 GHz MMIC VCO. This VCO shows excellent phase noise together with large tuning bandwith.

The paper is organized as follows: Sec. II presents derivation and motivation of the loop-gain and loaded-Q definition and its application to oscillator analysis. Sec. III discusses the results of this analysis. Finally, circuit design and measurements for a VCO are treated in Sec. IV.

II. Loop Gain and Loaded Q

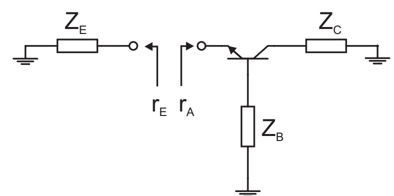

Fig. 1. Reflection-type oscillator.

Fig. 1 illustrates the basic circuit of a reflection-type oscillator. It consists of an active device, terminated by three impedances to ground. In the following a GaAs HBT will be treated as an example. Separating the circuit in two parts at the emitter, the small-signal oscillation condition reads:

$$r_E r_A = 1 \tag{1}$$

To define loop gain, the circuit as shown in Fig. 2 is examined. The amplifying part of the oscillator is reduced to the

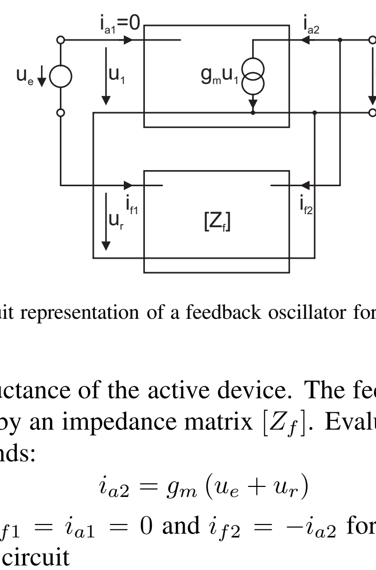

Fig. 2. Circuit representation of a feedback oscillator for loop-gain calculation.

transconductance of the active device. The feedback part is described by an impedance matrix $[Z_f]$. Evaluating the circuit one finds:

$$i_{a2} = g_m\left(u_e + u_r\right) \tag{2}$$

and with $i_{f1} = i_{a1} = 0$ and $i_{f2} = -i_{a2}$ for the feedback part of the circuit

$$u_r = Z_{f11} i_{f1} + Z_{f12} i_{f2} = -Z_{f12} i_{a2} \tag{3}$$

The overall transconductance g'_m is determined as

$$g'_m = \frac{i_{a2}}{u_e} = \frac{g_m}{1 + Z_{f12} g_m} \tag{4}$$

which corresponds to the common equations of oscillator analysis in control theory. The denominator of (4) is called the characteristic equation and the loop gain S_v is defined as

$$S_v = Z_{f12} g_m = |S_v|\, e^{j\varphi} \tag{5}$$

The loaded quality factor Q_L of the circuit is defined as the phase slope of the loop gain S_v:

$$Q_L = \left.\left(\frac{\omega}{2}\left|\frac{d\varphi}{d\omega}\right|\right)\right|_{\omega\,\rightarrow\,\omega_0} \tag{6}$$

One should note that this understanding of Q_L provides a direct relationship with phase noise, but, depending on loop-gain definition, it may differ from the physical definition of circuit-element Q factor. Rearranging the reflection-type os-

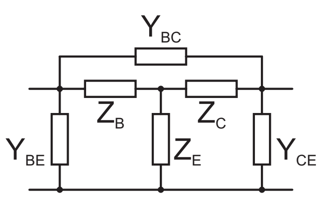

Fig. 3. Feedback path of reflection type oscillator in the configuration of Fig. 2.

cillator circuit of Fig. 1 with the emitter as the new ground node, one obtains a topology equivalent to Fig. 2. The three impedances Z_B, Z_C, and Z_E then form a T network with the former ground as center node. Except for the transconductance, all remaining elements of the transistor can be represented by a Π configuration in the feedback path as shown in Fig. 3 (Y_{BE},Y_{BC},and Y_{CE}). Using general admittance parameter Y_a for the active device, the loop gain S_v can be expressed analytically by

$$\begin{aligned} S_v &= \frac{(Z_E - Y_{a12} Z_x)(Y_{a21} - Y_{a12})}{1 + Z_x Y_x + Z_E Y_y + Y_{a11} Z_B + Y_{a22} Z_C} \quad \text{with} \\ Y_x &= Y_{a11} Y_{a22} - Y_{a12}^2 \qquad Y_y = Y_{a11} + Y_{a22} + 2Y_{a12} \\ Z_x &= Z_B Z_C + Z_B Z_E + Z_C Z_E \end{aligned} \tag{7}$$

As shown in Fig. 3, the resonator- or feedback path of a reflection-type oscillator consists of three internal admittances of the active device together with the three external impedances Z_B, Z_C, and Z_E. To investigate how the Q of the three external impedances translates into the loaded Q_L of the oscillator, equation (7) is analyzed under the assumption of the small-signal oscillation condition. To ensure oscillation start-up, instead of $S_v = -1$, the following condition for the left side of (7) has to be satisfied:

$$S_v = |S_v|\, e^{-j\pi}, \qquad \text{with}\, |S_v| > 1 \tag{8}$$

Because all of the three external impedances are connected to ground, they can be described by reflection coefficients easily. With Z_0 denoting the reference impedance, one has, e.g., at the emitter port:

$$r_E = \frac{Z_E - Z_0}{Z_E + Z_0} = |r_E|\, e^{j\varphi_E} \tag{9}$$

With magnitude and phase of the three external impedances and the loop-gain magnitude $|S_v|$ eqns. (7) and (8) involve a total of seven unkowns. To reduce the number of variables in the calculation, a reasonable restriction is to assume equal reflection-coefficient magnitudes for the three external impedances:

$$|r_E| = |r_B| = |r_C| = |r_i| \tag{10}$$

The phase of the emitter reflection-coefficient φ_E is treated as independent variable. Then, with $|r_i|$ and $|S_v|$ as parameters, the phase of the base and collector reflection coefficients φ_B and φ_C are used to fulfil eqns. (7) and (8). For each emitter phase φ_E one obtains a set of two solutions for base and collector phase φ_B and φ_C (for the detailed mathematics see Appendix A).

The phases of the external branches are described by an angle φ_0 plus a term adding a phase slope, which allows to vary the Q. For example, the base reflection coefficient is written as

$$r_B = |r_i| e^{j\varphi_B} = |r_i| e^{j\left(\varphi_{B0} + (\omega-\omega_0)\left.\frac{d\varphi}{d\omega}\right|_{\omega\rightarrow\omega_0}\right)} \tag{11}$$

In this way, the loop gain in eqn. (7) and, consequently, the loaded Q_L of the oscillator (see eqn. (8)) can be calculated.

The procedure can be summarized as follows:
1) Evaluate the admittance parameters Y_a of the active device at the desired frequency.
2) Fix the magnitude of the reflection coefficients of the three external impedances $|r_i|$ and the magnitude of the loop gain $|S_v|$ to the desired value.
3) For a given emitter phase φ_E, calculate the corresponding set of base and collector phases $\varphi_{B1,2}$ and $\varphi_{C1,2}$.
4) Add a phase slope to each reflection coefficient (in analogy to what is shown in equation (11) for the base).
5) Calculate the loaded Q of the oscillator.
Step 3) to 5) are carried out for each emitter phase in the range $\varphi_E = 0 \ldots 2\pi$.

III. Design Considerations

As an example, the performance of a reflection-type oscillator at 38 GHz with a two finger 3x30μm^2 GaInP/GaAs

HBT as active device is investigated. The small signal equivalent circuit according to [3] is used to describe the HBT. The pad capacitances in the HBT model are connected to ground, so they have to be stripped off before the calculation. For the magnitude of the reflection coefficients of the three external impedances $|r_i|$=0.98, for the magnitude of $|S_v|$=1.3 are assumed.

Three different cases are considered. In the first one, the base reflection coefficient is expanded by a phase-slope term $d\varphi/d\omega$=-5e-12 as shown in equation (11). φ_E and φ_C are constant with frequency and do not contribute to the Q_L of the oscillator. Then, Q_L is determined taking into account

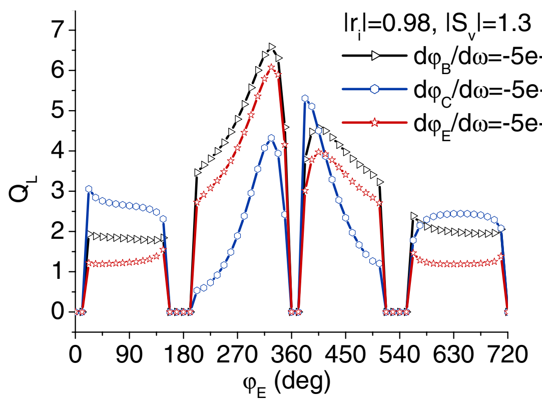

Fig. 4. Loaded Q of a 38 GHz reflection-type oscillator against emitter phase, for increased phase slope at base, collector, and emitter, respectively.

the inner elements of the HBT and the phase slope associated with the base impedance. In the second and third simulations, the phase-slope term is added to collector and emitter, respectively, while the other two impedances are assumed to be frequency independent in the vicinity of the oscillation frequency. In Fig. 4, the results are plotted. For emitter phases $\varphi_E \leq 2\pi$, the first solution set φ_{B1} and, φ_{C1}, for $\varphi_E > 2\pi$, the second solution pair φ_{B2} and φ_{C2} is used. Around emitter-phase values of $\varphi_E = n\pi$, oscillation is impossible and the calculated Q_L is set to zero.

With comparable conditions at the three HBT ports, maximum oscillator Q_L is achieved for an emitter phase of $\varphi_E = 327°$ (using the first solution of equation (15)). The corresponding phases at base and collector are $\varphi_B = 49°$ and $\varphi_C = 337°$ (all in deg). More generally, one concludes from Fig. 4 that the three external impedances do not influence Q_L equally. The phase-slope of the base impedance yields the highest Q_L.

In a second simulation, the effect of the magnitude $|r_i|$ of the external impedances is studied. For this purpose, we consider only the maximum of the curves in Fig. 4 as function of $|r_i|$. Fig. 5 presents the results. We observe a clear characteristic: Increasing the magnitude, Q_L of the oscillator increases as well. This can be explained by the fact that,

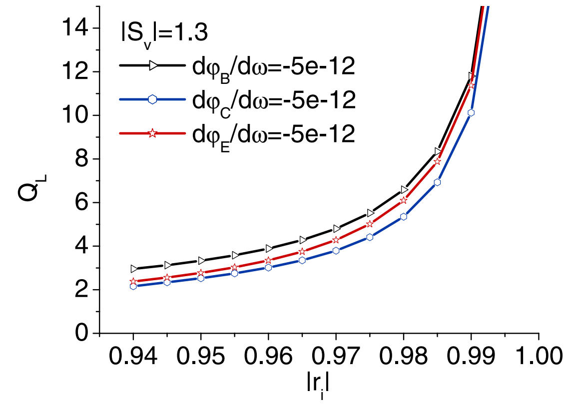

Fig. 5. Maximum Q_L of a 38 GHz reflection-type oscillator as a function of the magnitude $|r_i|$ of the external impedances, for increased phase slope at base, collector, and emitter, respectively.

for constant phase slope, a larger magnitude is equivalent to a larger Q factor of the impedances.

IV. Circuit Design and Realization

In order to verify the concept, the results described above are used to design a 38 GHz MMIC VCO. First, we identified the region of optimum phase values based on the curves in Fig. 4. In a second step, we designed three subcircuit versions, which allow to adjust the optimum phase values for each of the three ports. The networks consists of ground connections or block capacitances transformed by coplanar lines and spiral inductors, which are used as $\lambda/4$-transformers. At the base, the varactor is added.

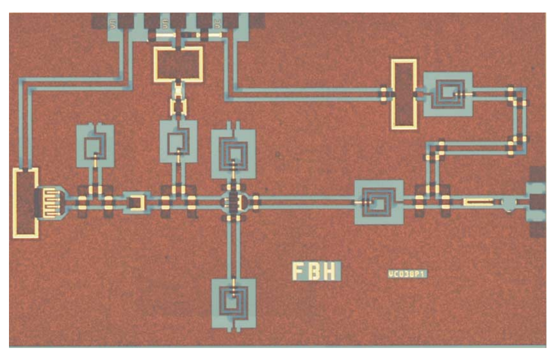

Fig. 6. Chip foto of the VCO (chip size is 2.1 x 1.3 mm^2).

Finally, fine tuning of the reflection coefficients in phase and magnitude was performed by adjusting the lengths of the connecting coplanar lines. We found the best performance (simulated values) with phases of $\varphi_E = 299°$, $\varphi_B = 91°$ and $\varphi_C = 265°$ and magnitudes of $|r_E|$=0.983, $|r_B|$=0.931 and $|r_C|$=0.975. This justifies the assumption of almost equal magnitudes eq. (10) is based on.

Fig. 6 shows a chip photo. The circuit is not optimized with regard to size. To reject load-pull effects during on-wafer measurements, a 10 dB on-chip attenuator is integrated at the output. Phase noise is measured with the E5504 system of Agilent applying the delay-line method, the results are plotted in Fig. 7. Because the varactor is strongly coupled to the base of the HBT, a large tuning bandwith of 2.4 GHz (= 6.8 %) at an oscillation frequency of 35.5 GHz is achieved. Nevertheless, the phase noise performance is excellent with -85 dBc/Hz at an offset frequency of 100 kHz. Considering the 10 dB on-chip attenuator, the actual power shows a smooth behavior in the range -1 to -5 dBm.

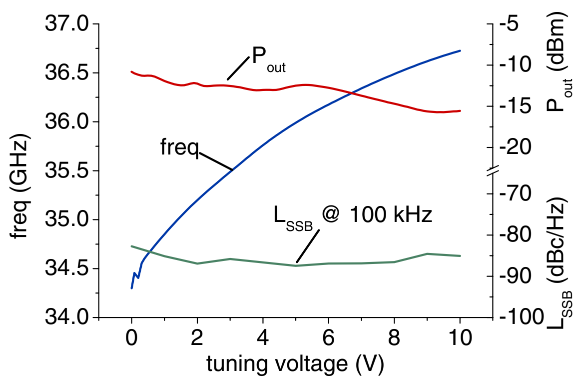

Fig. 7. Measurement results of the 38 GHz MMIC VCO: frequency, output power (behind the 10 dB attenuator), and SSB phase noise against tuning voltage.

V. Conclusions

A new approach for optimizing low phase-noise reflection-type oscillators is presented. It is especially suited for MMIC oscillators and allows to determine optimum conditions for the three impedances connected to the ports of the active device. This provides a mean to take maximum advantage of the element Q factors with respect to phase noise. A Ka band VCO was designed and realized to verify the approach. It achieves excellent phase noise performance together with a large tuning bandwidth.

VI. Appendix A

To solve (7) under condition of (8), a graphical procedure is used. The collector reflection coefficient $r_C = |r_i|\, e^{j\varphi_C}$ is part of a circle with the center in the origin of the smith chart. Because of the conformal mapping from S to Z parameters this results in a circle with center Z_{C0} and radius R_C in Z parameters:

$$Z_{C0} = Z_0 \frac{1+|r_i|^2}{1-|r_i|^2} + j0 \qquad R_C = \frac{2|r_i|Z_0}{|r_i|^2-1} \qquad (12)$$

In the next step, equation (7) is solved for $Z_B = f(Z_C)$:

$$Z_B = b_0 \frac{Z_C + b_1}{Z_C - b_2} \quad \text{with} \qquad (13)$$

$$\begin{aligned} Y_y &= Y_{a11} + Y_{a22} + 2Y_{a12} \\ Y_w &= Y_{a12}^2 - Y_{a12}Y_{a21} - S_v(Y_{a11}Y_{a22} - Y_{a12}^2) \\ b_0 &= \frac{S_v Y_{a22}}{Y_w} - Z_E \qquad b_2 = \frac{S_v Y_{a11}}{Y_w} - Z_E \\ b_1 &= \frac{Z_E(Y_{a21} - Y_{a12} - S_v Y_y) - S_v}{Z_E Y_w - S_v Y_{a22}} \end{aligned}$$

This represents a conformal mapping from Z_C to Z_B, so that Z_B is also described by a circle with center Z_{B0} and radius R_B. These parameters can be found by inserting three points of Z_C from equation (12) in (13):

$$\begin{aligned} Z_{B0} &= \frac{-b_0\left(R_C^2 + (b_1 + Z_{C0})(b_2 - Z_{C0})^*\right)}{|b_2 - Z_{C0}|^2 - R_C^2} \\ R_B &= \frac{|R_C b_0 (b_1 + b_2)|}{|b_2 - Z_{C0}|^2 - R_C^2} \end{aligned} \qquad (14)$$

A second condition for Z_B is that it also has to be part of a circle according to equation (12). Therefore, Z_B is part of two circles and one finds none, one, or two intersection points as solution:

$$Z_{B1,2} = Z_{B0} + \frac{Z_{C0} - Z_{B0}}{2Z_{BC}^2}\Big\{(R_B^2 - R_C^2 + Z_{BC}^2) \pm \sqrt{\left((R_B - Z_{BC})^2 - R_C^2\right)\left((R_B + Z_{BC})^2 - R_C^2\right)}\Big\}$$
$$\text{with} \quad Z_{BC} = |Z_{B0} - Z_{C0}| \qquad (15)$$

The solution for Z_C can be found by solving (13) for Z_C and inserting the result of (15):

$$Z_{C1,2} = \frac{b_0 b_1 + b_2 Z_{B1,2}}{Z_{B1,2} - b_0} \qquad (16)$$

In equation (15), only Z_{B0} and Z_{C0} are complex values. The difference of these two complex numberes in the numerator of (15) can be seen as a vector from Z_{B0} to Z_{C0}. The intersection of the two circles is located on a line normal to this vector. This happens, when the root expression is negative and leads to imaginary values. If the root expression is positive, the circles do not intersect and no solution is possible.

References

[1] D. B. Leeson, “A Simple Model of Feedback Oscillator Noise Spectrum,” *Proc. of IEEE*, vol. 54, pp. 329 – 330, 1966.

[2] J. C. Nallatamby, M. Prigent, M. Camiade, J. J. Obregon, “Extension of the Leeson formula to phase noise calculation in transistor oscillators with complex tanks,” *IEEE Trans. Microwave Theory and Techniques*, vol. 51, pp. 690 – 696, 2003.

[3] F. Lenk, M. Rudolph, “New Extraction Algorithm for GaAs-HBTs With Low Intrinsic Base Resistance,” *MTT-S Int. Microwave Symp. Dig.*, vol. 2, pp. 725 – 728, 2002.